Presuppositional implicatures: quantity or maximize presupposition?¹

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Abstract. Schlenker (2012) proposes that when framed within a modern Stalnakerian view of presupposition and common ground (Stalnaker, 1998, 2002), *Maximize Presupposition!* (Heim, 1991; Sauerland, 2008) can be viewed as a special case of the maxim of Quantity (Grice, 1975). We provide data suggesting that in some cases, *Maximize Presupposition!* applies even when speakers are not expected to use a presupposition as vectors of new information. We argue that these data support the view that *Maximize Presupposition!* is an independent pragmatic principle, distinct from Quantity.

Keywords: maximize presupposition, quantity, presuppositional implicatures, scalar implicatures.

1. Introduction

Much current discussion in pragmatics has been concerned with *Maximize Presupposition!* (Heim, 1991; Percus, 2006; Chemla, 2008; Sauerland, 2008; Schlenker, 2012), a rule of conversation proposed to account for the infelicity of certain utterances in contexts where a presupposition absent from them is felicitous. More specifically, we say that an utterance F is infelicitous if there exists some presuppositionally stronger alternative F' whose presupposition p is appropriate within the context. Such a statement will be clearer once the notions of *presuppositional alternative, presuppositional strength* and *presuppositional appropriateness* are properly defined.

In section 2, we present an overview of *Maximize Presupposition!* and the so-called presuppositional implicatures it predicts (Leahy 2016). Section 2.1 discusses the principle as it has classically been described (Heim, 1991; Percus, 2006; Sauerland, 2008), *viz.* as predicting how the use of presuppositionally weak alternatives will generate the inference that the presuppositions of their stronger alternatives are not common belief. In section 2.2, we discuss Chemla's (2008) arguments that adopting a modern Stalnakerian view of presupposition and common ground (Stalnaker, 1998, 2002) can account for the stronger inferences one gathers from the use of certain presuppositionally weak alternatives. In section 2.3, we discuss Schlenker's (2012) arguments that within this framework, one can understand presuppositional implicatures as following from the maxim of Quantity (Grice, 1975) rather than from an independent principle such as *Maximize Presupposition!*.

In section 3, we discuss problems with the proposals of Chemla and Schlenker. In section 3.1, we note that the notion of *authority*, introduced by Chemla to implement a modern Stalnakerian

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view, is too strong and predicts a number of unattested inferences. We propose to restrict his account by introducing the notion of speaker reliability. In section 3.2, we discuss how this notion makes different predictions depending on whether one treats *Maximize Presupposition!* as an independent principle or as a special case of Quantity. We offer data suggesting that it favors treating the principle as independent. In an appendix, which is not essential to our arguments, we spell out a proof of a result that is assumed in Chemla (2008).

2. Previous accounts of presuppositional implicatures

2.1. Maximize Presupposition!

In order to define the notion of presuppositional alternative, we must first define the set of *presuppositional scales* (Percus, 2006). Much like the scales used to define alternatives in neo-Gricean accounts of scalar implicature (Horn, 1972; Gazdar, 1979), this set will consist of a list of given pairs of lexical items. Here, we assume the set to contain exactly three elements, *viz.* the pairs $\langle a(n), the \rangle$, $\langle all, both \rangle$ and $\langle believe, know \rangle$.²

(1) Presuppositional scales The set of presuppositional scales $\Sigma_{\pi} = \{ \langle a(n), the \rangle, \langle all, both \rangle, \langle believe, know \rangle \}$

A given utterance will be a presuppositional alternative to another whenever both utterances differ syntactically only with respect to the substitution of one member from a scale for another member of that scale.

(2) Presuppositional alternatives F' is a presuppositional alternative to F, written as Alt_{π}(F', F), iff F' is identical to Fsave for the substitution of one member of a scale in Σ_{π} for another of that same scale.

We say that F' is presuppositionally stronger than F whenever the set of worlds in which F is neither true nor false strictly entails the set of worlds in which F' is neither true or false.³

(3) Presuppositional strength F' is presuppositionally stronger than F, written as $F' \prec_{\pi} F$, iff $\{w \in W : F = \#\} \subset \{w \in W : F' = \#\}$

The scales assumed above have been laid out in such a way as to ensure that the substitution of the rightmost element of a given scale for its leftmost element results in a presuppositionally stronger alternative. Indeed, we will assume that the extensions of the members of any given scale are identical save for an added presupposition in the item on the right. The table below (Marty, 2017) displays for each scale what the added presupposition of the rightmost item is.

²See Rouillard and Schwarz (2017) for an account of presuppositional alternatives which dispenses with scales and opts instead for a complexity based account to alternatives modeled on that of Katzir (2007) for scalar implicatures. ³One might argue that another important condition on some F' being presuppositionally stronger than some F would be that both share the same asserted content. While this is certainly true, the scales assumed here make stating this condition unnecessary for our purposes.

weak item	strong item	differential presupposition
a	the	uniqueness of the complement
all	both	domain contains exactly two of the complement
believe	know	the complement is true

We will for the moment assume that an utterance F' is presuppositionally appropriate whenever for any proposition p presupposed by an utterance of F', p is common belief. The notion of common belief is defined relative to the set of beliefs of the speaker s and her addressee a. Assuming the operator B_i to signify '*i believes* ...', we can define the set \mathscr{B} of higher-order beliefs of s and a according to the recursive definition in (4) (Stalnaker, 2002; Chemla, 2008; Schlenker, 2012).

- (4) (i) $\forall i \in \{s, a\}, B_i \in \mathscr{B}$
 - (ii) $\forall B, B' \in \mathscr{B}, BB' \in \mathscr{B}$
 - (iii) Nothing else is in \mathscr{B}

Using this definition for \mathcal{B} , we can now define what it means for a proposition p to be common belief.

(5) Common Belief

A proposition *p* is common belief, written as C[p], iff for every B in \mathcal{B} , B[p] = 1.

For an utterance F' to be presuppositionally appropriate, it must be the case that each of its presuppositions be common belief. That is, for any given p presupposed by F', it must be the case that $B_s[p]$, $B_a[p]$, $B_sB_a[p]$, $B_aB_s[p]$, $B_sB_s[p]$, $B_aB_a[p]$, ad infinitum.

(6) Presuppositional appropriateness F' is presuppositionally appropriate, written as $App_{\pi}(F')$, iff for all *p* presupposed by F', C[p]

A formal definition of *Maximize Presupposition!* (MP) can now be given in (7), which takes a form similar to that of a conversational maxim.

(7) Maximize Presupposition!
A speaker *s* must not utter some *F* if there is an *F'* such that *s* believes that:
(i) Alt_π(*F'*, *F*)
(ii) *F'* ≺_π *F*(iii) App_π(*F'*)

The literature on presuppositions reports the infelicity of examples such as those in (8a-10a) to be attributable MP (Heim, 1991; Singh, 2011).

- (8) a. #An independence of the United States is celebrated in July.
 - b. The independence of the United States is celebrated in July.

- (9) a. #Mary believes that 2+2=4.
 - b. Mary knows that 2+2=4
- (10) a. #John opened all his eyes at the same time.
 - b. John opened both his eyes at the same time.

Given the scales assumed above and the extensions assumed for their members, it follows that the *b* examples are presuppositionally stronger alternatives of the *a* examples, meaning that (7i) and (7ii) are met for MP. Moreover, in any normal context, the presupposition of the *b* examples will be common ground, ensuring that (7iii) is also met. Hence, the infelicity of the *a* examples is straightforwardly captured by the definition of MP in (7).

More than simply predict the infelicity of utterances who have presuppositional alternatives appropriate in all normal contexts, MP also predicts that one will draw inferences whenever the presuppositionally weaker of two alternatives is employed (Percus, 2006; Sauerland, 2008). Indeed, it will follow from the utterance of a weak presuppositional alternative that the speaker does not believe the utterance of its stronger counterpart to have been appropriate. According to the definition of appropriateness assumed so far, this will lead to the inference that the speaker does not believe that the presupposition of the stronger alternative is common belief. Such *presuppositional implicatures* (PIs) are illustrated by the examples in (11-13).

- (11) John is looking for the number of a girl he met in Berlin. PI: $\neg B_s C[$ *that John met exactly one girl in Berlin*]
- (12) All of the papers Mary submitted were rejected. PI: $\neg B_sC[$ *that Mary submitted exactly two papers*]
- (13) John believes that Mary is pregnant. PI: $\neg B_s C[that Mary is pregnant]$

Ascertaining whether such inferences are in fact drawn from the examples in (11-13) is a difficult task due in no small part to how weak the predicted inferences are. Indeed, for it not to be the case that *s* believes that *p* is common belief, it need only be the case that for some arbitrary B in \mathscr{B} , $\neg B_s B[p]$. Thus for example, it will not be the case that *s* takes *p* to be common belief in cases ranging from her believing *p* to be false, believing that *a* takes *p* to be false, believing that *a* is unsure about the truth of *p*, being unsure herself of the truth of *p*, believing that *a* does not believe *s* to believe *p* to be true, and so on. Certainly the weakness of such an inference casts doubt on the value of its prediction by MP, as any attempt to test for the presence of such an inference seems entirely hopeless.

2.2. Authority (Chemla 2008)

Chemla (2008) notes that the notion of presuppositional appropriateness discussed in (6) is too weak to capture the inferences one intuitively gathers from the utterance of certain presuppositionally weak alternatives. Indeed, what one infers from an utterance of the examples in (14-16) is not simply that *s* does not take the presupposition of their stronger alternatives to be common

ground, but rather that *s* herself does not believe the presupposition of these alternatives to be true.

- (14) A bathroom in my apartment is flooded.⁴ Predicted PI: $\neg B_s C[$ *that there is exactly one bathroom in s's apartment*] Actual PI: $\neg B_s[$ *that there is exactly one bathroom in s's apartment*]
- (15) All my brothers fought in Vietnam. Predicted PI: $\neg B_s C[$ *that s has exactly two brothers*]Actual PI: $\neg B_s[$ *that s has exactly two brothers*]
- (16) John believes that I have a sister. Predicted PI: $\neg B_s C[that \ s \ has \ a \ sister]$ Actual PI: $\neg B_s[that \ s \ has \ a \ sister]$

Chemla proposes to solve this problem by transitioning to a modern Stalnakerian view of presupposition and common ground (Stalnaker, 1998, 2002). Under this account, Stalnaker defines presuppositional appropriateness similarly to how it was defined in (6), meaning that for a speaker to presuppose p is appropriate implies that $B_sC[p]$. However, the innovation in this account is that appropriateness is defined not as requiring p to be common belief prior to its presupposition by s, but after it has been presupposed. The driving force behind this idea is that if after p's presupposition a comes to believe p, then it will follow that C[p]. In order to address this issue, we refer to Chemla's proposal that an epistemic step is involved in the derivation of PIs, which appeals to the notion of *authority*. A speaker s is an authority relative to a and with respect to some presupposition p whenever s presupposing p will cause a to *accommodate* and believe p. More generally, authority can be viewed as a special case of the assumption that s is correct in her beliefs, and this by assuming that whenever s presupposes p, she is committed to the truth of p. To this effect, we adopt Schlenker's (2012) formalization of authority below.

(17) Authority $B_a[B_s[p] \Rightarrow p]$

A concept such as *authority* offers a new way of describing presuppositional appropriateness. In order for some F' to be presuppositionally appropriate, the maxim of Quality (Grice, 1975) requires a cooperative speaker to believe every presupposition he makes when uttering F'. However, rather than require that some p be common belief prior to its presupposition by s, all that is needed is for s to be an authority on p such that p becomes common belief following s's presupposition of p.⁵

(18) Presuppositional appropriateness App $_{\pi}(F')$ iff for all *p* presupposed by F', $B_s[p] \wedge B_a[B_s[p] \Rightarrow p]$

Now consider once again the examples in (14-16) in light of our new notion of presuppositional

⁴This example was devised by Michael Wagner, (p.c.)

⁵See the appendix for a discussion on how the notion of presuppositional appropriateness in (18) paired with the assumption that *s* is an authority on some presupposition *p* is sufficient to guarantee that a presupposition *p* becomes common belief following its utterance.

appropriateness. For *s* to utter these presuppositionally weak alternatives will cause *a* to infer that *s* does not believe that uttering their stronger alternatives is appropriate $(\neg B_s(B_s[p] \land B_a[B_s[p] \Rightarrow p]))$. In other words, from an utterance of weaker alternatives, *a* will derive the PI that either *s* does not believe *p* or that *s* does not believe that she is an authority on *p*.

(19) Presuppositional implicature $\neg B_s[p] \lor \neg B_s B_a[B_s[p] \Rightarrow p]$

The epistemic step Chemla proposes in order to obtain the inferences observed in (14-16) relies on the interaction between the predicted disjunctive PIs in (19) and what he dubs the *Authority Assumption* (AA). Simply put, the AA is an assumption made by a whereby she assumes that sbelieves herself to be an authority on p.

(20) Authority assumption $B_s B_a[B_s[p] \Rightarrow p]$

With our new definition of presuppositional appropriateness and the AA, it becomes easy to see how one obtains from (14-16) their attested inferences. Let F be any of these utterances and F' be its presuppositionally stronger alternative such that F' presupposes p but F does not. MP predicts that an utterance of F by s will lead a to draw the PI in (19). However, in these cases, a assumes that s believes herself to be an authority on p. As a result, the inference drawn from F can be strengthened such that what a concludes from its utterance is that s does not believe p.

(21) Left Side PI strengthening

$$(\neg B_s[p] \lor \neg B_s B_a[B_s[p] \Rightarrow p]) \land (B_s B_a[B_s[p] \Rightarrow p]) \vDash \neg B_s[p]$$

Chemla's account makes a further prediction, *viz*. that whenever it is clear that *s* believes *p*, any PI regarding *p* will be strengthened on the right-side, *i.e.* the PI will be strengthened such that what is entailed is that *s* does not believe herself to be an authority on *p*.

(22) Right Side PI strengthening

$$(\neg B_s[p] \lor \neg B_s B_a[B_s[p] \Rightarrow p]) \land B_s[p] \vDash \neg B_s B_a[B_s[p] \Rightarrow p]$$

Chemla argues that the example in (23) provides evidence that right side strengthening does indeed appear where predicted. (23) competes with a presuppositionally stronger alternative, leading to the PI in (19). However in this utterance, *s* clearly states that she believes Mary is pregnant. Chemla's account therefore predicts that from an utterance of (23), *a* will infer that *s* does not believe herself to be an authority on Mary being pregnant. Chemla claims that this is the intuitive reading one obtains from (23), but problems with this analysis will be discussed in section 3.1.

(23) I believe that Mary is pregnant. Predicted PI: $\neg B_s B_a [B_s[that Mary is pregnant] \Rightarrow that Mary is pregnant]$

2.3. Maximize Presupposition! as Quantity (Schlenker 2012)

Schlenker (2012) notes the parallel between the drawing of PIs from presuppositionally weak alternatives and the drawing of scalar implicatures within a neo-Gricean framework. He attempts to reduce MP as an independent principle to Gricean reasoning by proposing that the conversational principle according to which one must always use the presuppositionally stronger of two alternatives follows from the need to be as informative as possible. In other words, Schlenker proposes to reduce MP to Quantity, and as such reduce PIs to scalar implicatures.⁶

Schlenker makes use of Chemla's notion of authority to account for how presuppositions can be informative in a context where a does not believe p. Assuming s to be an authority on p, her uttering p will result in a believing p. In such cases, presupposing p therefore seems to be a means of transmitting p as new information. Thus, in a context where s believes p and believes that she is an authority on p, her using the weaker of two presuppositional alternatives can be interpreted as a violation of Quantity (Grice, 1975), as the presuppositionally stronger alternative would have been more informative. From the point of view of a, the reasoning follows very closely that of scalar implicatures. Assume that a does not believe p but makes the AA. If s uses the presuppositionally weak F rather than its stronger alternative F', a will reason that if s believed p, her failure to use F' would result in a violation of Quantity. Therefore a will infer that s, who is taken to be cooperative, does not believe p.⁷ Following Schlenker, one can propose a definition of *informativity* which states that an utterance F' is more informative than an utterance F whenever it is presuppositionally stronger than F or strictly entails F.

(24) Informativity F' is more informative than F, written as $F' \prec F$, iff $\{w \in W : F = \#\} \subset \{w \in W : F' = \#\}$ or $\{w \in W : F' = 1\} \subset \{w \in W : F = 1\}$

In order to propose a general pragmatic principle which equates PIs to scalar implicatures, it will also be necessary to extend the notions of *alternatives* and *appropriateness*. The first step in accomplishing this is to define a set of scales which includes not only presuppositional scales, but also scales relevant to scalar implicatures, in this case $\langle some, all \rangle$ and $\langle or, and \rangle$.

(25) Scales The set of scales $\Sigma = \{ \langle a(n), the \rangle, \langle all, both \rangle, \langle believe, know \rangle, \langle some, all \rangle, \langle or, and \rangle \}$

⁶Leahy (2016) pursues the same approach, but for reasons of space we confine attention to Schlenker's execution of the idea.

⁷As noted in the literature (Heim, 1991; Percus, 2006), MP does not hold only in situations where *a* is assumed to not believe *p*, but crucially also holds when *p* is common belief prior to its presupposition by *s*. To account for these cases, Schlenker introduces the idea that there exists parallel to any given common ground a weakened common ground where it is not common belief that *p*, and this on account of the small chance that *a* will have forgotten *p*. Through a mechanism Schlenker calls *recoverability*, such weakened common grounds can be updated following a presupposition of *p* by *s*, ensuring that even when *p* is already common belief, it will be informative insofar as it updates the weakened common ground.

This extended set of scales can be used to define a set of alternatives which can therefore be used both for the computing of scalar implicatures as well as what has so far been assumed to be PIs.

(26) Alternatives F' is an alternative to F, written as Alt(F',F), iff F' is identical to F save for the substitution of one member of a scale in Σ for another member of that same scale.

Finally, we can extend the notion of appropriateness by stating that F' is appropriate if both its presupposed and assertive contents are believed by *s* and if *s* is an authority on both.

(27) Appropriateness F' is appropriate, written as App(F'), iff for all p presupposed or asserted by F', $B_s[p] \wedge B_a[B_s[p] \Rightarrow p]$

With these notions in hand, we can now propose a general pragmatic principle, *Be Informative!* (BI), according to which speakers should not use some utterance F if there exists some F' which is an alternative to F, more informative than F and appropriate.

(28) Be Informative!

A speaker *s* must not utter some *F* if there is an F' such that *s* believes that:

- (i) $\operatorname{Alt}(F, F')$
- (ii) $F' \prec F$
- (iii) $\operatorname{App}(F')$

3. Authority and reliability

3.1. Problems With Authority

Let us for the moment set aside possible reductions of MP to Quantity and return to a framework where the two principles are disjoint. Consider once again Chemla's prediction in (22), where he claims that in a sentence like (23), restated below, a will infer that s is not an authority on Mary being pregnant.

(23) I believe that Mary is pregnant. Predicted PI: $\neg B_s B_a [B_s[that Mary is pregnant] \Rightarrow that Mary is pregnant]$

Chemla purports that what one intuitively gathers from (23) is that *s* is not an authority about Mary being pregnant, but it is not so clear that this is truly the inference one draws from that sentence. Recall that *authority* in its technical sense is defined as *a*'s willingness to accommodate a presupposition *p* if *s* believes *p*. To say that (23) yields the inference that *s* does not believe she is an authority about Mary being pregnant implies that *s* does not believe that, had she presupposed that Mary is pregnant, *a* would not have accommodated this presupposition. This seems far too strong an inference for what one intuitively gathers from (23), *viz*. that *s* does not believe she is a reliable source of information regarding whether or not Mary is pregnant. In fact, the contrast between (29a) and (29b) may provide further evidence that the reliability

of the speaker is important to the computing of PIs. Consider the difference between the PI in (29a) and (29b).

- (29) a. John believes that I have a sister. PI: $\neg B_s[that \ s \ has \ a \ sister]$
 - b. John believes that Mary has a sister. Unattested PI: $\neg B_s[that Mary has a sister]$

It is far from clear that from (29b) one can infer very much about s's beliefs on whether or not Mary has a sister. Indeed, the contrast between the inference drawn from (29a) and that of (29b) can be made sharper if one considers whether or not it is acceptable for a to question the inference. As noted by Marty (2017), PIs can be disputed using the *Hey, wait a minute!* test first discussed by von Fintel (2004). We report our judgments that while it is fine for a to call into question s having a sister following an utterance of (29a), it is odd for a to question Mary having a sister following (29b). This may provide further evidence that the reliability of s regarding the presupposition of an utterance's alternative is important to whether or not one strengthens the PI. While in (29a) it seems reasonable to assume that s is a reliable source of information regarding whether or not she has a sister, one assumes that in (29b), s is not reliable regarding whether or not Mary has one.⁸

(30)	a.	s: John believes that I have a sister.	
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- a: Hey, wait a minute! You don't have a sister?
- b. *s*: John believes that Mary has a sister.
 - *a*: #Hey, wait a minute! Mary doesn't have a sister?

There is in fact good reason to believe that defining presuppositional appropriateness in terms of *s*'s beliefs on *p* and on whether she is an authority on *p* provides an account of MP which is much too strong. Consider once again the examples in (11-13), and consider the failure of the *Hey, wait a minute!* test on these.

(31)	a.	s: John is looking for the number of a girl he met in Berlin.
		<i>a</i> : #Hey, wait a minute! John met more than one girl in Berlin?

- b. *s*: All of the papers Mary submitted were rejected. *a*: #Hey, wait a minute! Mary submitted more than two papers?
- c. s: John believes Mary is pregnant.
 - *a*: #Hey, wait a minute! Mary isn't pregnant?

Compare these results with those we obtain when considering the examples in (14-15).

(32) a. s: A bathroom in my apartment is flooded.a: Hey, wait a minute! There's more than one bathroom in your apartment?

⁸In fact, the strong inference derived from (29a) does not appear in contexts where *s* is not reliable on whether she has a sister. Consider its utterance in a context where *s* is an orphan, and has been told by some acquaintance that he recalls her adoption papers mentioning that *s* had a sister. Here, one would not draw from (29a) the inference that *s* does not believe she has a sister.

b. s: All of my brothers fought in Vietnam.a: Hey, wait a minute! You have more than two brothers?

For Chemla's account of PIs to not predict strengthened PIs from the utterances in (11-13), it would have to be the case that for each of these, the AA is not made by a. But this is once again a highly questionable premise as there is no question that, baring disagreement, a would accommodate the presuppositions of the stronger alternatives of each of these sentences. Why then would a not assume that s believes herself to be an authority on these presuppositions? To argue that this is what one concludes from the data would be to set the stage for a circular argument, and what one wants here is not to simply state the facts, but to offer an explanation for them. What seems necessary is to strengthen our notion of presuppositional appropriateness so as to weaken our PIs. As noted above, s's reliability seems to play an important role regarding whether or not PIs are strengthened, and would thus serve as a good candidate to strengthen appropriateness. Of course, even when s is unreliable regarding some p, if a already believes p, then it will be appropriate for s to presuppose p so long as s also believes p. Hence, presuppositional appropriateness can be strengthened in (33) by adding to its definition that it must either be the case that s is reliable about p or that a already believes p.

(33) Presuppositional appropriateness App_{π}(*F'*) iff for all *p* presupposed by *F'*, B_{*s*}[*p*] \wedge B_{*a*}[B_{*s*}[*p*] \Rightarrow *p*] \wedge (B_{*a*}[*p*] \vee Rel(*s*, *p*)), where Rel(*s*, *p*) is to be read as '*s* is reliable about *p*'

Trivially, whenever *a* already believes *p*, it follows that *s* is an authority on *p*. From this, it is easy to see that for *s* to be an authority on *p* and for *a* to already believe *p* is equivalent to simply saying that *a* believes *p*. From this result, we can show that our definition of presuppositional appropriateness is equivalent to the one in $(34)^{10}$.

(34) Presuppositional appropriateness (equivalent formula) $B_s[p] \land (B_a[p] \lor (B_a[B_s[p] \Rightarrow p] \land \text{Rel}(s, p)))$

Now imagine that *s* is not reliable with respect to some presupposition *p*. In such a situation, it follows that $(B_a[B_s[p] \Rightarrow p] \land \text{Rel}(s, p))$ is false, in which case, $(B_a[p] \lor (B_a[B_s[p] \Rightarrow p] \land \text{Rel}(s, p))$ is equivalent to simply $B_a[p]$. In such contexts, a presupposition would be appropriate only when both *s* and *a* believe *p*.

(35) Presuppositional appropriateness (when *s* is not reliable on *p*) $B_s[p] \wedge B_a[p]$

¹⁰This can be shown by the following reasoning: $B_s[p] \land (B_a[B_s[p] \Rightarrow p] \land (B_a[p] \lor \operatorname{Rel}(s, p))) \equiv$ $B_s[p] \land ((B_a[B_s[p] \Rightarrow p] \land B_a[p]) \lor (B_a[B_s[p] \Rightarrow p] \land \operatorname{Rel}(s, p))) \models$ $B_s[p] \land (B_a[p] \lor (B_a[B_s[p] \Rightarrow p] \land \operatorname{Rel}(s, p)))$

⁹See Rouillard and Schwarz (2017) for arguments that surprisal and even speaker efficiency also play a role in determining whether a presupposition is appropriate. What seems plausible is that appropriateness should be strengthened by the conjunction of a series of disjuncts, among which would be reliability, the addressee's beliefs in p, surprisal and efficiency. For the sake of simplicity and clarity, we assume here only reliability and the addressee's belief.

Assuming that s is not reliable with respect to p in (11-13), we predict the PI for each of these utterances to be the formula in (36).

- (36) Presuppositional Implicature (when *s* is not reliable on *p*) $\neg B_s[p] \lor \neg B_s B_a[p]$
- 3.2. Presuppositional implicatures from Quantity? Comparing Predictions

Let us now return to the reduction of MP to Quantity discussed in section 2.3. Much like the version of MP in section 2.2, this account relies heavily on *authority* in order to show how presuppositions could be used to update contexts. As discussed, this will run into problems when considering the examples in (11-13) as, barring disagreement, it is hard to imagine why s would ever use the weaker alternative of some F' presupposing p. Consider the vantage point of a for any of these utterances when assuming that speakers are expected to obey BI as stated in (28). Assuming a does not already believe p (but does not believe p to be false), a will reason following these utterances that there exists for each of them a more informative alternative F'. From this, a will infer that either s does not believe the presuppositions of F' or does not believe herself to be an authority on them. As discussed earlier, there is no reason for a not to make the AA, as it is a matter of common sense that she would have accommodated the presuppositions, in which case the inferences predicted from (11-13) will be that s does not believe the presuppositions of their alternatives. As discussed above, these predictions are inaccurate. A natural move to make here would be to amend appropriateness in BI in the same way it was amended for MP in section 3.1, viz. by restricting appropriateness with the disjunction of reliability and addressee belief in p.

(37) Appropriateness App(F') iff for all p presupposed or asserted by F', $B_s[p] \wedge B_a[B_s[p] \Rightarrow p] \wedge (B_a[p]) \vee Rel(s,p))$

However, it is easy to see that such a formulation of appropriateness is far too strong. Consider once again the utterance in (29b), stated in (38a), as well as the very similar utterance in (38b).

- (38) a. John believes that Mary has a sister.
 - b. John believes that Jane has a sister.

We assume that in each of these cases, the weak PIs obtained are the result of s being unreliable with respect to the presuppositions of their alternatives, *i.e.* s is unreliable on Mary having a sister and unreliable on Jane having a sister. But now consider the example in (39a) in a context where a does not know about whether Mary or Jane have siblings, which competes with the alternative in (39b).

(39) a. Mary has a sister or Jane has a sister. Inference: ¬B_s[Mary has a sister and Jane has a sister]
b. Mary has a sister and Jane has a sister.

Given our assumption that s is not reliable on Mary having a sister and Jane having a sister, one would predict for (39a) an inference on par with that in (36). That is, one would predict from (39a) the inference in (40).

(40) $\neg B_s[$ *that Mary has a sister and Jane has a sister* $] \lor \neg B_s B_a[$ *that Mary has a sister and Jane has a sister*]

This is of course not what one intuitively gathers from (39a), from which speakers infer (in addition to ignorance inferences) that it is not the case that both Mary and Jane have a sister. One could attempt a further restriction on appropriateness such that it applies solely to presuppositions, as in (41).

(41) Appropriateness App(F') iff for all p presupposed or asserted by F', $B_s[p] \wedge B_a[B_s[p] \Rightarrow p]$ and for all q such that q is presupposed, $B_a[q] \lor Rel(s,q)$

Such a notion of appropriateness, however, runs into an important conceptual problem if one tries to reconcile it with treating presuppositions as informative. Consider once more a context in which s is not reliable on some presupposed p. The notion of appropriateness when considering the presupposition p will be the one in (35), restated below.

(42) Appropriateness (when s is not reliable on p) $B_s[p] \wedge B_a[p]$

This suggests that, were s to believe p to be true but not believe that a takes p for granted, presupposing p would be judged inappropriate by s. For s to judge p to be inappropriate on account of a not already knowing p seems to run counter to the idea that presuppositions are to be understood as vectors of new information. The question becomes how to maintain the distinction between (29a) and (29b), where (29a) seems to generate an inference similar to a scalar implicature while (29b) does not, while nevertheless preventing appropriateness from taking the form in (42). One solution is to remove any mention of a's beliefs from the conditions on presuppositional appropriateness. That is, rather than have these conditions be the disjunction $(B_a[p] \lor Rel(s, p))$, these can be simply stated as Rel(s, p). This would however appear to be too strong a notion of appropriateness. Indeed, this would predict that it is inappropriate for s to ever presuppose some proposition p on which s is not a reliable source of information. We know, however, that p will always be appropriate when it is already taken for granted by both conversational partners, and this irrespective of whether or not s is reliable on p. Faced with such a problem, it would appear that modifying the notion of appropriateness is incompatible with an account of MP which treats presuppositions as informative. The soundest move from here would be to redefine our notion of informativity. That is, we will assume that unless s is reliable on p, p cannot be informatively used as a presupposition.

(43) Informativity

- $F' \prec F$ iff,
- (i) $\{w \in W : F = \#\} \subset \{w \in W : F' = \#\}$ or $\{w \in W : F' = 1\} \subset \{w \in W : F = 1\}$
- (ii) For every *p* presupposed by an utterance of F', Rel(s, p)

Let us now assess what predictions our amended version of BI makes when *s* utters the weaker of two alternatives *F* such that *s* is not reliable on the presupposition *p* of the stronger alternative *F'*. Given that *s* is not reliable on *p*, it will follow from our definition of informativity that *F'* is not more informative than *F*. As a result, *s* is not expected to use the stronger alternative and, thus, no inference is predicted from her utterance of *F*. We now have a clear difference in the predictions of MP as an independent principle and BI. When *s* is unreliable on the presupposition *p* of *F'*, MP predicts that an utterance of *F* will generate the inference in (44). On the other hand, BI predicts that no inference will be generated from such an utterance.

$$(44) \qquad \neg \mathbf{B}_{s}[p] \lor \neg \mathbf{B}_{s} \mathbf{B}_{a}[p]$$

Of course, the inference in (44) is extremely weak, and it is unclear whether one could ever report perceiving such an inference from the utterance of some weak presuppositional alternative. However, following Chemla's idea of an epistemic step for MP, we can verify whether this disjunctive inference is strengthened in contexts where *a* assumes one of the disjuncts to be false. For instance, if *a* assumes *s* to believe that *p*, an utterance by *s* of some weak *F* competing with an *F'* presupposing some *p* (for which *s* is unreliable) will be predicted to yield the inference in (44) which, given *a*'s beliefs, will be strengthened to simply $\neg B_s B_a[p]$. To test this, consider an utterance of (11), within a context where *a* knows that John met exactly one girl in Berlin and is certain that *s* is also aware of this.¹¹ The judgment is subtle, but seems correct. If a speaker were to utter (11) when we know very well that she knows John met exactly one girl, we would infer that she takes us, as addresses, to be unaware of this fact. This intuition can be reinforced by considering the felicity of the dialog in (45), where *a* calls attention to *s*'s use of the weaker alternative.

(45) s: John is looking for the number of a girl he met in Berlin.a: Hey, wait a minute! A girl he met in Berlin? We both know he met one girl there.

The same test can be applied to (12) and (13). In (12), we assume a to be certain about s knowing that Mary submitted exactly two papers while in (13), a is certain about s knowing that Mary is pregnant.

- (46) s: All of the papers Mary submitted were rejected.a: Hey, wait a minute! All of the papers Mary submitted? We both know she submitted two.
- (47) s: John believes that Mary is pregnanta: Hey, wait a minute! John *believes* that Mary is pregnant? We both know that she is.

¹¹We require that a be certain that s is aware of this fact in order to prevent a from revising her beliefs on s's belief that John met exactly one girl in Berlin.

Clearer judgments are perceptible when (44) is strengthened by assuming the right-hand disjunct is false. This can be achieved by having *a* assert (or presuppose) *p*, only to have *s* respond to *a* by using the weaker *F* rather than the presuppositionally stronger *F'*. In this case, we predict the inference in (44) to be strengthened such that what is infered is that *s* does not believes *p*, $(\neg B_s[p])$.

(48)	a.	<i>a</i> : Is John looking for the number of the girl he met in Berlin?
		s: John is looking for the number of a girl he met in Berlin.
	b.	<i>a</i> : Whatever happened to the two papers Mary submitted?

- s: All of the papers Mary submitted were rejected.
- c. *a*: Did you hear the news from John? He just told me Mary is pregnant. *s*: John believes that Mary is pregnant.

In all of these cases, *s*'s avoidance of the presuppositionally stronger alternative generates the predicted inference. Crucially, this is not predicted from BI, as the presupposition of the alternative is not taken to be informative on account of *s*'s lack of reliability.

4. Conclusion

This paper argues that a challenge to attempts at reducing presuppositional implicatures to scalar implicatures arises once it is recognized that *authority* in and of itself is insufficient to account for such inferences. Indeed, a principle such as BI, even when enriched by the notion of reliability, does not predict weak inferences from the utterances in (11-13). On the other hand, a principle such as MP independent of notions of informativity seems not only able to predict these inferences, but moreover predicts the epistemic strengthening operated on examples (14-16). It would appear as though the imperative to presuppose as much as possible is not fully explicable in terms of informativity. Rather, speakers must sometimes reason not only about what is accommodatable in the common ground, but also about what is common ground prior to their utterances. That is, speakers are not expected to use presuppositions for which they are not reliable unless these are already taken for granted by them and their addressee.

Appendix

The modern Stalnakerian view of presupposition and common ground argues that a presupposition is appropriate if it becomes common belief after its utterance that p. To this effect, Schlenker (2012) assumes that when s presupposes p, it becomes common belief that s believes p will be common belief at some time t at which a checks the presupposition p. With this in mind, he proves that at t, if $CB_sC[p]$ is true and a has indeed accommodated p ($B_a[p]$), it follows that C[p]. We show here that the definition of presuppositional appropriateness in (18) paired with the assumption that s is an authority on p will be sufficient to ensure that p is common belief after it is presupposed by s, thus deriving the results of Schlenker's proof without the need to assume that s presupposing p leads to inferences about s's beliefs on the future. In order to prove this, we must first introduce the lemma in (49).

(49) Lemma 1 $\forall i [\mathbf{B}_i[p] \Leftrightarrow \mathbf{B}_i \mathbf{B}_i[p]]$

We follow Stalnaker (2002) in assuming that beliefs are represented by an accessibility relation R_i such that $B_i[p]$ is true if and only if for all worlds satisfying w R_i w', *p* is true in w'. We further assume that R_i is transitive, euclidean and serial.¹²

- (50) a. Transitivity: $\forall w \forall w' \forall w'' [w Riw' \land w' R_i w'' \Rightarrow w R_i w'']$
 - b. Euclideanity: $\forall w \forall w' \forall w'' [w R_i w' \land w R_i w'' \Rightarrow w' R_i w'']$
 - c. Seriality: $\forall w \exists w' [w R_i w']$

Assume that $B_i[p]$ is true in w. Then p is true in all worlds w' satisfying w R_i w'. By transitivity, it follows that all worlds w'' satisfying w' R_i w'' also satisfy w R_i w'', and thus that p is true in all such worlds. From this, we can conclude that in all w' satisfying w R_i w', $B_i[p]$ is true, and thus it must be the case that $B_iB_i[p]$ is true in w. In other words, for all *i*, if $B_i[p]$, then $B_iB_i[p]$.

Assume that $B_i B_i[p]$ is true in the world of evaluation w for some arbitrary *i*. Then, for all worlds w' satisfying wR_iw', it will be the case that $B_i[p]$, and in all worlds w'' satisfying w'R_iw'', it will be the case that *p*. Given that R_i is transitive, it follows that all worlds w'' satisfying w'R_iw'' also satisfy wR_iw''. Hence in all such worlds $B_i[p]$ holds. Given euclideanity, all worlds w' satisfying wR_iw' must also satisfy w''R_iw', and hence in all such worlds it must be the case that *p*. Thus, in all worlds w' satisfying wR_iw', it must be the case that *p*, and therefore it must be the case that $B_i[p]$ in w. This in turn entails that if $B_iB_i[p]$ is true in w, then so is $B_i[p]$.

Having shown that for all *i*, if $B_i[p]$, then $B_iB_i[p]$ and if $B_iB_i[p]$, then $B_i[p]$, we conclude that for all *i*, $B_i[p]$ is true if and only if $B_iB_i[p]$. QED

The second lemma we introduce will be that whenever the common ground entails that it is common belief that $B_s[p]$ and it is common belief that $B_a[p]$, it will be common belief that p.

(51) Lemma 2 If (i) $CB_s[p]$ (ii) $CB_a[p]$ then (iii) C[p]

Assume that both $CB_s[p]$ and $CB_a[p]$ are true.

C[p] is true according to our definition of common belief in (4) if and only if for all B in the set \mathscr{B} , B[p] is true. This entails on the one hand that both $B_s[p]$ and $B_a[p]$ are true and on the other that for any sequence S of two or more belief operators, S[p] is also true.

¹²While seriality is not essential to our proofs, it does simplify them by allowing us to disregard all cases where there is no w' satisfying w R_i w'.

If $CB_s[p]$, then it follows by our definition of common belief that $B_sB_s[p]$. We can conclude from (49) that because $B_sB_s[p]$, then $B_s[p]$. Hence, it follows from $CB_s[p]$ that $B_s[p]$. If $CB_a[p]$, then it will be the case that $B_aB_a[p]$ according to our definition of common belief, and from (49) we can conclude that $B_a[p]$. It therefore follows that if $CB_s[p]$ and $CB_a[p]$, then $B_s[p]$ and $B_a[p]$.

Let S be an arbitrarily chosen sequence of two or more belief operators from \mathscr{B} . Then it is either the case that S ends in B_s or in B_a .

Case 1: Assume S ends in B_s . Then S can be represented as the concatenation S' B_s of some non-empty sub-sequence S' of S and B_s . Clearly, S' is a sequence of at least one belief operator. Given our assumption that $CB_s[p]$, it follows by our definition of common belief that S' $B_s[p]$ is true, and thus that S[p] is also true.

Case 2: Assume S ends in B_a . Then once again S is the concatenation S' B_a of some non-empty sub-sequence S' and B_a . Once again, S' is a sequence of belief operators and thus it follows from our assumption that $CB_a[p]$ that S' $B_a[p]$, and therefore that S[p].

We can conclude from this that for any sequence S of two or more operators, S[p] holds if both $CB_s[p]$ and $CB_a[p]$ do. This in addition to the fact that $B_s[p]$ and $B_a[p]$ follow from $CB_s[p]$ and $CB_a[p]$ allows us to conclude that if $CB_s[p]$ and $CB_a[p]$ are true, then for all $B \in \mathcal{B}$, B[p] is true. This in turn entails by our definition of common belief in (4) that C[p] is also true. QED

Following Stalnaker (2002), we assume that s's speech act of presupposing p is a manifest event, *i.e.* an event which ensures that after it occurs it will be common belief that it has occurred. Hence, when s presupposes p, it becomes common belief that s believes p is appropriate, or equivalently, it becomes common belief that s believes p and common belief that s believes she is an authority on p.

(52)
$$\operatorname{CB}_{s}[p] \wedge \operatorname{CB}_{s}\operatorname{B}_{a}[\operatorname{B}_{s}[p] \Rightarrow p]]$$

Let us assume that *s* presupposes *p* at some time *t*. As a result of this speech act, it becomes common belief at t+1 that *s* believes that it is appropriate to presuppose *p*, in which case it follows that (52) is true. If *s* is in fact an authority on *p*, *i.e.* if *a* is willing to accommodate *p* when *s* believes *p*, then it follows that *p* is common belief.

If (i) $CB_s[p]$ (ii) $CB_sB_a[B_s[p] \Rightarrow p]$ (iii) $B_a[B_s[p] \Rightarrow p]$ then (iv) C[p]

(53)

Assume that $CB_s[p]$, $CB_sB_a[B_s[p] \Rightarrow p]$ and $B_a[B_s[p] \Rightarrow p]$ are all true.

Consider all possible sequences of members of \mathscr{B} that can precede B_a in $CB_a[p]$. B_a can be preceded by a sequence with only instances of B_s , a sequence with only instances of B_a , or a sequence S containing both instances of B_s and B_a .

Case 1: Let B_s^n be a sequence of *n* instances of B_s , where $n \in \mathbb{N}$. Given that $CB_sB_a[B_s[p] \Rightarrow p]$, it follows that $B_sB_sB_a[B_s[p] \Rightarrow p]$ is true, which by (49) entails that $B_sB_a[B_s[p] \Rightarrow p]$. Given that $CB_s[p]$, it follows that $B_sB_aB_s[p]$. $B_sB_a[B_s[p] \Rightarrow p]$ and $B_sB_aB_s[p]$ together allow us to conclude that $B_sB_a[p]$, which we can rewrite as $B_s^1B_a[p]$. Now let there be some arbitrary $m \in \mathbb{N}$ such that $B_s^mB_a[p]$ is true. By (49), it follows that $B_s^mB_a[p]$, in which case we can conclude by mathematical induction that for all $m \in \mathbb{N}$, $B_s^mB_a[p]$, and hence we conclude that $B_s^nB_a[p]$ is true.

Case 2: Let B_a^n be a sequence of *n* instances of B_a , where $n \in \mathbb{N}$. Given that $CB_s[p]$, we know that $B_aB_s[p]$. Paired with our assumption that $B_a[B_s[p] \Rightarrow p]$, this entails that $B_a[p]$. Through the same reasoning as in case 1, it follows that for all $n \in \mathbb{N}$, $B_a^n[p]$ is true.

Case 3: Let S be a sequence of B_s and B_a . Then either S is the concatenation $S'B_a^1B_s^n$ of some (possibly empty) sub-sequence S' of S, one instance of B_a and some arbitrary sequence of *n* instances of B_s where $n \in \mathbb{N}$, or S is the concatenation $S'B_s^1B_a^n$, where $n \in \mathbb{N}$.

Case 3.1: Assume S is the concatenation $S'B_a^1B_s^n$. Given our assumption that $CB_sB_a[B_s[p] \Rightarrow p]$ is true, it follows that $S'B_a^1B_s^{n-1}B_sB_a[B_s[p] \Rightarrow p]$. Likewise, given that $CB_s[p]$ is true, so must be $S'B_a^1B_s^{n-1}B_sB_aB_s[p]$. Together, these entail that $S'B_a^1B_s^{n-1}B_sB_a[p]$, or equivalently that $S'B_a^1B_s^nB_a[p]$.

Case 3.2: Assume S is the concatenation $S'B_s^1B_a^n$. Given that $CB_sB_a[B_s[p] \Rightarrow p]$ is true, so must be $S'B_sB_sB_a[B_s[p] \Rightarrow p]$, which by (49) is equivalent to $S'B_sB_a[B_s[p] \Rightarrow p]$. Given that $CB_s[p]$ is true, it follows that $S'B_sB_aB_s[p]$ is also true. Together, these entail that $S'B_sB_a[p]$, which can be rewritten as $S'B_s^1B_a^1[p]$. Assume that $S'B_s^1B_a^m[p]$ is true for some arbitrarily chosen *m* such that $m \in \mathbb{N}$. Given (49), it follows that $S'B_s^1B_a^{m+1}[p]$, and thus by mathematical induction, for all $m \in \mathbb{N}$, $S'B_s^1B_a^m[p]$ is true. We can thus conclude that $S'B_s^1B_a^{n+1}[p]$ is true, or in other words, that $S'B_s^1B_a^nB_a[p]$ is true.

We see that for any sequence S of members of \mathscr{B} , $SB_a[p]$ is true, and thus by our definition of common belief, $CB_a[p]$ must be true. Since both $CB_s[p]$ and $CB_a[p]$ are true, by (51) it follows that C[p]. QED

An important point noted by Chemla is that in case of a disagreement on a given proposition p, it will not be the case that s is an authority on p. We let the reader convince herself that if s is not an authority on p, it will not be the case that an utterance of p by s will make p common belief. A further point to note is the fact that in cases where a already believes p, s's authority on p is trivially met. Here too we let the reader convince herself that if a already believes p at the moment of its utterance by s, p will be common belief following this utterance.

References

- Chemla, E. (2008). An epistemic step for anti-presuppositions. *Journal of Semantics* 25(2), 141–173.
- Gazdar, G. (1979). *Pragmatics: Implicature, Presupposition and Logical form.* New York: Academic Press.
- Grice, P. (1975). Logic and conversation. In P. Cole and J. L. Morgan (Eds.), *Syntax and Semantics. 3: Speech Acts*, pp. 41–58. New York: Academic Press.
- Heim, I. (1991). Artikel und Definitheit. In A. von Stechow and D. Wunderlich (Eds.), *Semantik: ein internationales Handbuch der Zeitgenossischen Forschung*, pp. 487–535. Berlin: Mouton de Gruyter.
- Horn, L. (1972). On the semantic properties of logical operators in English. Ph. D. thesis, University of California, Los Angeles.
- Katzir, R. (2007). Structurally-defined alternatives. *Linguistics and Philosophy 30*(6), 669–690.
- Leahy, B. (2016). On presuppositional implicatures. *Topoi* 35(1), 83–91.
- Marty, P. P. (2017). *Implicatures in the DP domain*. Ph. D. thesis, Massachusetts Intitute of Technology, Cambridge.
- Percus, O. (2006). Antipresuppositions. In A. Uyema (Ed.), *Theoretical and Empirical Studies of Reference and Anaphora: Toward the Establishment of Generative Grammar as an Empirical Science*, pp. 52–73. Japan Society for the Promotion of Science.
- Rouillard, V. and B. Schwarz (2017). Epistemic narrowing from maximize presupposition. In A. Lamont and K. Tetzloff (Eds.), *Proceedings of the 47th annual meetung of the North East Linguistic Society (NELS 47)*, pp. 42–62. GLSA, Department of Linguistics, University of Massachusetts.
- Sauerland, U. (2008). Implicated presuppositions. In A. Steube (Ed.), *The Discourse Potential* of Underspecified Structures. Berlin: Mouton de Gruyter.
- Schlenker, P. (2012). Maximize Presupposition and Gricean reasoning. *Natural Language Semantics* 20(4), 391–429.
- Singh, R. (2011). Maximize presupposition! and local contexts. *Natural Language Semantics* 2(19), 149–168.
- Stalnaker, R. C. (1998). On the representation of context. *Journal of Logic, Language, and Information* 7(1), 3–19.
- Stalnaker, R. C. (2002). Common ground. Linguistics and Philosophy 25(5), 701–721.
- von Fintel, K. (2004). Would you believe it? the king of france is back! presuppositions and truth-value intuitions. In M. Reimer and A. Bezuidenhout (Eds.), *Descriptions and beyond*, pp. 315–341. Oxford University Press.